Noncommutativity and Lorentz Violation in Relativistic Heavy Ion Collisions

P. Castorina[1,2], A. Iorio[3], D. Zappalà[2]

[1] *Dipartimento di Fisica, Università di Catania, Via Santa Sofia 64, I-95123 Catania, Italy.*
[2] *INFN, Sezione di Catania, I-95123 Catania, Italy.*
[3] *Faculty of Mathematics and Physics, Charles University in Prague*
V Holešovičkách 2, 18000 Prague 8 - Czech Republic

(Dated: April 15, 2010)

Abstract

The experimental detection of the effects of noncommuting coordinates in electrodynamic phenomena depends on the magnitude of $|\theta B|$, where θ is the noncommutativity parameter and B a background magnetic field. With the present upper bound on θ, given by $\theta_{\rm bound} \simeq 1/(10\ \text{TeV})^2$, there was no large enough magnetic field in nature, including those observed in magnetars, that could give visible effects or, conversely, that could be used to further improve $\theta_{\rm bound}$. On the other hand, recently it has been proposed that intense enough magnetic fields should be produced at the beginning of relativistic heavy ion collisions. We discuss here lepton pair production by *free* photons as one kind of signature of noncommutativity and Lorentz violation that could occur at RHIC or LHC. This allows us to obtain a more stringent bound on θ, given by $10^{-3}\ \theta_{\rm bound}$, if such "exotic" events do not occur.

PACS numbers: 25.75.-q, 25.75.Dw, 25.75.Nq

High energy heavy-ion collisions are the most important tool to investigate the behavior of quantum chromodynamics (QCD) at large temperature and density. The analysis of QCD phase diagram has required a great theoretical effort and the initial understanding in terms of a phase transition from confined quarks and gluons to a weakly interacting quark-gluon plasma (QGP) has been deeply modified by the experimental results at the Relativistic Heavy Ion Collider (RHIC). The new scenario of a strongly interacting QGP will be definitively clarified by future experiments at the Large Hadron Collider (LHC).

On the other hand, it has been recently shown [1–3] that other fundamental aspects of QCD, related to the topological nature of its vacuum, can be directly tested by experimental observations in relativistic heavy ion collisions. Indeed, gluon field configurations with non zero topological charge generate chiral asymmetry, inducing P and CP violating effects which produce an asymmetry between the amount of positive/negative charge above and below the reaction plane [1–3].

STAR Collaboration presented [4] the conclusive observation of charge-dependent azimuthal correlations, however the explanation of this charge asymmetry by $P-$ and CP-odd dynamics requires that a strong magnetic field is produced at the beginning of the collision [5]. Analytical calculations [2] and numerical simulations [6] show that it is possible to produce an extremely intense magnetic field $|B| \simeq m_\pi^2$ in peripheral heavy ion collisions at RHIC. What we want to stress here is that the production of magnetic fields of such intensity could open a window on the detection of fundamental properties of space-time through the manifestation of effects of the noncommutativity of coordinates [7] and the associated violation of Lorentz symmetry [8].

The literature on the various mechanisms for Lorentz violation is vast and includes the Standard Model Extension [9], theories with speed of light differing from c and various other string theory/quantum gravity-inspired effective field theories [10]. In all cases, though, the bounds on the Lorentz violating parameters make the experimental appreciation extremely elusive. For example, a low energy remnant of quantum gravity is posited to be a modification of the dispersion relation for the photon given by, ($c = 1$),

$$E^2 - k^2 = -\xi k^3/M_{Pl} \tag{1}$$

where $M_{Pl} \simeq 10^{19}$ GeV is the Planck mass and ξ

is a parameter that should be smaller than 10^{-15} [11].

Let us now turn our attention to Maxwell theory in a noncommutative spacetime where $x^\mu \star x^\nu - x^\nu \star x^\mu = i\theta^{\mu\nu}$, with $\star$ the Moyal-Weyl product, $\theta^{\mu\nu}$ an antisymmetric constant tensor (see, e.g., [12]) and $\mu, \nu = 0, 1, 2, 3$. The general recipe to deal with gauge theories in such spacetimes was given in [13]. We are interested here on the $O(\theta)$-corrected action for Maxwell theory given by

$$\hat{I} = -\frac{1}{4}\int d^4x\, [F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\theta^{\alpha\beta}F_{\alpha\beta}F^{\mu\nu}F_{\mu\nu} + 2\theta^{\alpha\beta}F_{\alpha\mu}F_{\beta\nu}F^{\mu\nu}], \quad (2)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and A_μ the *usual* Abelian gauge field. Clearly, noncommutative electrodynamics (NCED) is essentially a nonlinear generalization of Maxwell electrodynamics and it was shown [14] that plane waves exist and, while those propagating along the direction of a background magnetic field $\vec{B}$ still travel at the usual speed of light, those which propagate transversely to $\vec{B}$ have a modified dispersion relation given by

$$\omega = k(1 - \vec{\theta}_T \cdot \vec{B}_T) \quad (3)$$

where $\vec{\theta} \equiv (\theta^1, \theta^2, \theta^3)$ is the spatial part of $\theta^{\mu\nu}$ ($\theta^i = \frac{1}{2}\epsilon^{ijk}\theta^{jk}$, $i, j, k = 1, 2, 3$, the temporal components are taken to be zero $\theta^{0i} = 0$) and the subscript T indicates the transverse component with respect to $\vec{k}$. That the quantum theory of NCED is sound is still an open issue due to certain novel divergencies that might [15] or might not [16] appear in such theories, depending on the mathematical framework one uses for noncommutativity. Nonetheless, many phenomenological implications can be studied within the classical frame or by focusing on the kinematical bounds on quantum processes. With this approach NCED, where the breaking of Lorentz invariance is directly related to noncommutativity [17], has been discussed by considering synchrotron radiation [18], Čerenkov effect in vacuum [19], ultra high energy gamma rays [20]. In all these works it was concluded that the effects of a nonzero $\vec{\theta}$ in NCED are very hard to detect due to the actual upper bound $\theta_{\rm bound} \simeq 1/(10{\rm TeV})^2$ [8, 21] and to the unavailability in nature of intense enough magnetic fields.

According to our previous discussion, in the initial state of relativistic heavy ion collisions, for large impact parameters, the magnetic field at the center of the collision can be approximately written as, [2, 6, 22]

$$B(t) = \frac{1}{[1 + (t/\tau)^2]^{3/2}} B_0, \quad (4)$$

with $\tau = b/(2\sinh Y)$, $B_0 = 8Z\alpha_{\rm EM}\sinh Y/b^2$ where b denotes the impact parameter, Z the charge of the nucleus, and Y the beam rapidity. For Gold-Gold ($Z = 79$) collisions at RHIC (at 100 GeV per nucleon) one has $Y = 5.36$ and, at typical large impact parameters $b = 10$ fm, one finds $B_0 \sim 1.9 \times 10^5$ MeV^2 and $\tau = 0.05$ fm/c.

From here it appears that the initial magnetic field in relativistic heavy ion collisions is much larger than the magnetic fields of magnetars and one can consider the possibility of detection of the Lorentz violating nonzero $\vec{\theta}$ effects. With these extremely intense magnetic fields one gains for $|\vec{\theta}_T \cdot \vec{B}_T|$ more than 20 orders of magnitude with respect to magnetic fields in terrestrial laboratories.

Before the evaluation of the impact of such gain on the dispersion relations in Eq. (3), which will be our main point here, let us give first an estimate of the impact on the noncommutative corrections to the synchrotron radiation spectrum of an energetic quark traveling in the magnetic field produced at the beginning of the collision.

The synchrotron radiation spectrum in NCED has been evaluated in [18]. Let us call ω the radiation frequency, $\omega_0 \sim 1/|\vec{r}|$ the cyclotron frequency, $\omega_c = 3\omega_0\gamma^3$ the critical frequency, where $\vec{r}$ is the radius of the orbit and γ the Lorentz factor. In the range $1 << \omega/\omega_0 << \gamma^3$, the ratio of the θ-corrected energy I (radiated in the plane of the orbit at large distances from the origin, within an angle $d\Omega$) to the standard one $I_{\theta=0}$, in the most favourable case $\vec{\theta}_T || \vec{B}_T$, is given by

$$X \equiv \frac{dI(\omega)/d\Omega}{dI(\omega)/d\Omega|_{\theta=0}} \sim 1 + 20\left(\frac{\omega_0}{\omega}\right)^{2/3} |\theta B|\gamma^4 \,. \quad (5)$$

For light energetic valence quarks in the heavy ion beams of 100 GeV per nucleon the Lorentz factor could easily be $\gamma \simeq O(10^2 - 10^3)$ and by considering $\omega_0 \simeq 1/\tau$, where τ has been introduced in Eq.(4), the range $1 << \omega/\omega_0 << \gamma^3$ allows the production of high frequency radiation. With $|\theta B| \simeq 10^{-9}$, for Gold-Gold collisions at RHIC, the ratio X can be larger than 1 indicating that either the spectrum is the θ-corrected or, conversely, that the $\theta_{\rm bound}$ used must be ameliorated. We do not proceed further with this case because to go beyond a mere indication of the effect here we should consider the situation in greater detail (higher order contributions, time-varying magnetic fields, parton distribution of the initial nucleon momentum, etc.).

What we can instead reliably focus on here are the dispersion relations in Eq.(3) regarded as the kinematical threshold for event obviously forbidden in standard electrodynamics, i.e. the pair production from a *single free* photon $\gamma \to e^+e^-$. In Eq.(3) the noncommutative contribution depends on the angle between $\vec{B}_T$ and $\vec{\theta}_T$. At first order in θ one has

$$\omega(1 + \vec{\theta}_T \cdot \vec{B}_T) = k \tag{6}$$

or

$$E_\gamma^2 - k^2 = -2E_\gamma^2(\vec{\theta}_T \cdot \vec{B}_T). \tag{7}$$

where $E_\gamma \equiv \omega$.

Defining by p_γ, p_+ and p_- the four momenta of γ, of e^+ and of e^-, respectively, the kinematical condition for the decay of a free photon whose action is given by (2) is

$$-2E_\gamma^2(\vec{\theta}_T \cdot \vec{B}_T) = (p_+ + p_-)^2 > 4m_e^2 \tag{8}$$

which requires $(\vec{\theta}_T \cdot \vec{B}_T) < 0$. Therefore if in a heavy ion collision at RHIC one produces a magnetic field of about 10^5 MeV^2, with $\theta \simeq \theta_{\rm bound}$, a free photon can produce an e^+e^- pair. Hence an enhancement of lepton pairs with low invariant mass could be the signal of the "exotic" effects of noncommutativity. With a magnetic field $B \sim 2 \times 10^5$ MeV^2 a photon with energy $E_\gamma = 50$GeV opens a new channel for lepton pairs of invariant mass of about 2.5MeV, while for $B \sim 2 \times 10^6$ MeV^2 and $E_\gamma = 100$GeV the invariant mass is about 7.5MeV.

Only γs produced at the very beginning of the collisions have to be considered because the magnetic field rapidly decreases with time (see Eq.(4)) and noncommutative effects, if any, fade away accordingly. Those high energy γs should produce pairs of very small invariant mass (see Eq.(8)). This implies that the electron and positron are essentially produced collinearly but, since such "exotic" decay occurs at the very beginning of the collision, one should observe the e^+ and e^- with very large energy and an initial opposite curvature. Thus this pair production mechanism is completely different from: (a) the standard Drell-Yan process, which applies for large invariant masses; (b) resonances decay, because one is considering e^+e^- with invariant mass much smaller than the resonance masses; (c) the usual $\gamma\gamma \to e^+e^-$ and/or pair production in inhomogeneous electromagnetic field. The latter pairs, however, could produce a large background.

The first step to single out the effects of the magnetic field is to consider the ratio between the yields of low invariant mass lepton pairs in nucleus-nucleus (A-B) and proton-proton (p-p) collisions because in the latter the collective magnetic field is negligible [2]. However in heavy ion collisions rescattering effects still generate a significant background and therefore the detection of the γ decay allowed by noncommutative spacetime requires a more careful analysis.

Let us define the laboratory frame, say $(\hat{x}, \hat{y}, \hat{z})$, in such a way that the reaction plane corresponds to the $\hat{x} - \hat{y}$ plane so that the magnetic field B is produced in the $\hat{z}$ direction [2]. The noncommutative effect are enhanced if the magnetic field transverse to the direction of the momentum of the photon is maximum (see Eq.(8)) and then one has to focus on the reaction plane. In particular, if $\hat{y}$ is the beam axis, to avoid the background in the forward direction, it is convenient to consider the γ production in the reaction plane and with large transverse momentum. For instance, if $\vec{k} \simeq (k_x, 0, 0)$ the noncommutative effect depend on the product $\theta_z B$.

From this point of view a clear signal would be an enhancement, event by event, of pairs of low invariant mass in the reaction plane with respect to pairs produced outside the reaction

plane.

Moreover, the noncommutative parameter $\vec{\theta}$ is fixed in a non-rotating frame, denoted by $(\hat{X}, \hat{Y}, \hat{Z})$, whereas the component θ_z used above is defined in the previously introduced frame. Since this frame, at fixed $\vec{b}$, rotates with the earth, this component changes in time with the periodicity that depends on the earth's sidereal rotation frequency Ω. By following the choice in [8, 23], one can take the $\hat{Z}$ direction of the non-rotating frame coincident with the rotation axis of the earth and $\hat{X}$ and $\hat{Y}$ with specific fixed celestial equatorial coordinates. Then, by indicating with $(\theta_X, \theta_Y, \theta_Z)$ the components of the noncommutative parameter in the non-rotating frame, one gets the explicit time dependence of θ_z [23]

$$\begin{aligned} \theta_z \ = \ & (\sin\chi \cos\Omega t) \ \theta_X + \\ & (\sin\chi \sin\Omega t) \ \theta_Y + \cos\chi \ \theta_Z \end{aligned} \tag{9}$$

where χ is the non-vanishing time-independent angle between the two axes $\hat{Z}$ and $\hat{z}$. The oscillation in θ_z disappears in the peculiar case of $\vec{\theta}$ coincident with the earth rotation axis (i.e. $\vec{\theta} = \theta_Z$ and therefore $\theta_z = \cos\chi \ |\vec{\theta}|$) whereas it is maximal if $\vec{\theta}$ lies in the equatorial plane. Apart from the unlikely case $\vec{\theta} = \theta_Z$, Eq. (9) clearly shows the oscillating structure of the product $(\vec{\theta}_T \cdot \vec{B}_T)$ which appears in in Eq. (8) and which, for the photons considered above, reduces to the product $\theta_z B$.

Then a nonvanishing $\vec{\theta}$ induces a periodicity with frequency Ω in the number of pairs produced at fixed $\vec{b}$. In particular, for sufficiently high values of B and θ to fulfill the bound in Eq. (8), by looking at the ratio between pairs produced in (A-B) and in (p-p) collisions and by selecting those pairs generated by photons with high transverse momentum in the reaction plane, so to reduce the effect of the background, one should also be able to observe the periodic time dependence of this ratio, in accordance with Eq. (9).

If, on the contrary, the ratio of pair production number in (A-B) and (p-p) shows no time dependence, one can nonetheless establish a new bound on the components of $\vec{\theta}$ associated with the oscillating factors, i.e. θ_X and θ_Y, based solely on the kinematical features which characterize the decay of free γ into e^+e^-. At LHC for Gold-Gold collision the magnetic field could easily reach the intensity $B_0 \simeq 3.2$ GeV^2. If a free photon with an energy of $E_\gamma \simeq 100$ GeV travels in such magnetic field produced at the beginning of the collision and the pair production *is not observed*, then Eq. (8) gives the following bound

$$\theta_{X,Y} < \frac{2m_e^2}{BE_\gamma^2} \simeq \frac{1}{10^5(\text{TeV})^2} \simeq 10^{-3} \ \theta_{\text{bound}} \tag{10}$$

where θ_{bound} is the known bound [8].

Strictly speaking, Eq.(3) requires a constant background and one has to assume that pair production indeed happens for short time $t < \tau$ after the collision (see Eq.(4)).

To conclude: if extremely intense magnetic fields are produced in relativistic heavy ion collisions they might either help to observe effects of the noncommutative nature spacetime or, if such effects are not found, to establish a more stringent bound on θ. Of course, a more detailed analysis is required for a direct comparison with experiments, nonetheless the enhancement of lepton pair production at low invariant mass we are suggesting here is a good probe as it is directly related with this new physics.

Acknowledgements The authors thank D. Kharzeev for very useful discussions and suggestions.

[1] D.E. Kharzeev, Phys. Lett. **B**633 (2006) 260; D.E. Kharzeev and A.Zhitnitsky , Nucl. Phys. **A**797 (2007) 67;

[2] D.E. Kharzeev, L.D. McLerran and H.J. Warringa, Nucl. Phys. **A**803 (2008) 227.

[3] K.Fukushima, D.E. Kharzeev and H.J. Warringa, "Real time dynamics of the Chiral magnetic effect", arXiv:hep-ph 1002.2495.

[4] B.I. Abelev et al. (Star Collaboration) Phys. Rev. Lett. **103** (2009) 251601; B.I. Abelev et al. (Star Collaboration) arXiv: 0909.1717 [nucl-ex].

[5] For different explanations see : F.Wang, arXiv: 0911.1482 [nucl-ex]; R.Millo and E.Shuryak, arXiv: 0912.4894 [hep-ph]; A.Bzdak, V.Koch and J.Liao, arXiv: 0912.5050 [nucl-th];

[6] V.Skokov, A.Y.Illarionov and V.Toneev, Int. J. Mod. Phys.**A**24 (2009) 5925;

[7] M.R. Douglas, N.A. Nekrasov, Rev. Mod. Phys. 73 (2001) 977; R.J. Szabo, Phys. Rep. 378 (2003) 207; I. Hinchliffe, N. Kersting and Y.L. Ma, "Review of the phenomenology of noncommutative geometry", ArXive: hep-ph/0205040.

[8] S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane, and T. Okamoto, Phys. Rev. Lett. 87 (2001) 141601.

[9] D. Colladay, V. Kostelecky, Phys. Rev. D58 (1998) 116002.

[10] For a review see G.Amelino-Camelia, "A perspective in quantum gravity phenomenology", ArXive: gr-qc/0402009; T. Jacobson, S. Liberati, D. Mattingly , "Quantum gravity phenomenology and Lorentz violation", ArXive: gr-qc/0404067; F. W. Stecker, J. Phys. G 29, (2003) R47; T. Jacobson, S. Liberati, D. Mattingly, Nature 424 (2003) 1019.

[11] T. Jacobson, S. Liberati, D. Mattingly and F.W. Stecker, Phys. Rev. Lett. 93:021101 (2004)

[12] J. Madore, S. Schraml, P. Schupp and J. Wess Eur. Phys. J. **16**C (2000) 161.

[13] N. Seiberg and E. Witten, JHEP 9909 (1999) 032.

[14] Z. Guralnik, R. Jackiw, S.Y. Pi, A.P. Polychronakos, Phys. Lett. B 517 (2001) 450.

[15] S. Minwalla, M. van Raamsdonk, N. Seiberg, J. High Energy Phys. **0002** (2000) 020.

[16] A. P. Balachandran, A. Pinzul, B. A. Qureshi Phys. Lett. B **634** (2006) 434.

[17] A. Iorio, Phys. Rev. D 77 (2008) 048701; A. Iorio, T. Sykora, Int. J. Mod. Phys. A 17 (2002) 2369.

[18] P. Castorina, A. Iorio and D. Zappalà, Phys. Rev. D69 (2004) 065008.

[19] P. Castorina, A. Iorio and D. Zappalà, Europhys. Lett.69 (2005) 912

[20] P. Castorina, and D. Zappalà, Eur. Phys. Lett. 64 (2003) 641; P. Castorina, A. Iorio, D. Zappalà, Nucl. Phys. **B** (Porc. Suppl.) 136 (2004) 333.

[21] For bound on noncommutative parameters in phase space see O.Bertolami et al. , Phys. Rev. D72 (2005) 025010.

[22] D.E. Kharzeev and H.J.Warringa, arXiv: hep-ph 0907.5007.

[23] V.A. Kostelecky and C. Lane, Phys. Rev. **D 60** (1999) 116010.